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108. $\Delta = \frac{L}{2\cos A \cos B \cos C}$, where L is the area of the pedal triangle or triangle formed by joining the feet of the perpendiculars.

109. $\Delta = M\frac{(a+b)(b+c)(c+a)}{2abc}$, where M is the area of the triangle formed by joining the feet of the internal bisectors.

110. $\Delta = \frac{2R}{r} . N$, where N is the area of the triangle formed by joining the points of tangency of the in-circle.

Concluding Note.—Owing to some changes and additions in the foregoing article made since its original preparation and while in press, the statement on page 135, Vol. I, that "the total number of formulæ * * * in this collection is *ninety-three*," etc., is no longer true.

The number of formulæ in the collection is shown below where the second column shows the number that result from counting as distinct those formulæ arising from permutation.

Group I,	31	31
Group II,	41	123
Group III,	11	44
Group IV,	10	60
Group V,	1	12
Miscellaneous Group,	16	18
Total,	110	288

[In the original manuscript all the forms arising from permuting the letters were given in full. To save space in printing, those forms which arose from mere cyclical permutation were omitted.—*O. S.*]

PROBLEM AND SOLUTION.

By Dr. J. E. Hendricks, Des Moines, Iowa.

Problem—To inscribe a rectangle whose sides are c,x in another rectangle whose sides are a,b, and express x in functions of a, b, and c.

SOLUTION.—Let $ABCD$ represent the rectangle a,b, and $MNPQ$ the rectangle c,x, and put angle $ANM =$ angle $BPN = \varphi$. Then is

$$x \cos\varphi + c \sin\varphi = a, \qquad (1)$$

$$x \sin\varphi + c \cos\varphi = b. \qquad (2)$$

From equations (1) and (2) we readily find

$$\cos\varphi = \frac{ax - bc}{x^2 - c^2}$$

$$\sin\varphi = \frac{bx - ac}{x^2 - c^2}$$

Squaring, adding, and reducing, we get

$$x^4 - (a^2 + b^2 + 2c^2)\,x^2 + 4abcx$$
$$- c^2(a^2 + b^2 - c^2) = 0. \qquad (3)$$

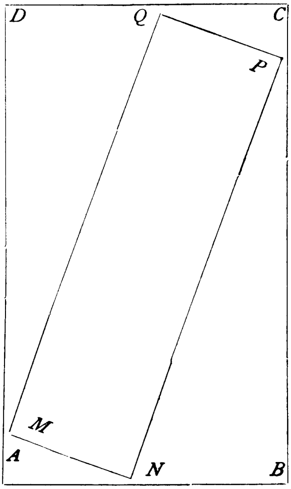
Putting, for brevity,

$$a^2 + b^2 + 2c^2 = m,$$

$$4abc = n;$$

and $$(a^2 + b^2 - c^2)\,c^2 = r,$$

(3) becomes $x^4 - mx^2 + nx - r = 0.$ (3′)



Transposing (3′) we have,

$$x^4 = mx^2 - nx + r.$$

Adding $2kx^2 + k^2$ to both sides of this equation it becomes

$$x^4 + 2kx^2 + k^2 = (m + 2k)\,x^2 - nx + (k^2 + r),$$

or $$x^2 + k = \sqrt{[(m + 2k)\,x^2 - nx + (k^2 + r)]}. \qquad (4)$$

Now, in order that the expression under the radical in (4) shall be an exact square, we must have

$$2\sqrt{(m + 2k)} \times \sqrt{(k^2 + r)} = n,$$

or $$(m + 2k) \times (k^2 + r) = \tfrac{1}{4}n^2.$$

Performing the multiplication indicated and transposing, we get

$$k^3 + \tfrac{1}{2}mk^2 + rk - (\tfrac{1}{8}n^2 - \tfrac{1}{2}mr) = 0. \qquad (5)$$

Equation (5) is a complete cubic involving k as the unknown quantity. In order to apply Cardan's formula to the solution of (5) we must first transform (5) by substituting $z - \frac{1}{6}m$ for k, by which it becomes

$$z^3 - (\tfrac{1}{12}m^2 - r)\,z - \tfrac{1}{2}n^2 + \tfrac{1}{108}m^3 + \tfrac{1}{3}mr = 0. \tag{6}$$

Putting $p = (\frac{1}{12}m^2 - r)$, $q = -\frac{1}{8}n^2 + \frac{1}{108}m^3 + \frac{1}{3}mr$

(6) becomes

$$z^3 - pz + q = 0. \tag{6$'$}$$

By applying Cardan's formula to (6′) we get

$$z = \left(-\frac{q}{2} + \sqrt{\frac{q^2}{4} - \frac{q^3}{27}}\right)^{\frac{1}{3}} + \left(-\frac{q}{2} - \sqrt{\frac{q^2}{4} - \frac{q^3}{27}}\right)^{\frac{1}{3}} \tag{7}$$

If we assume $a = 3$, $b = 2$, and $c = 1$, we find, by substitution, $p = 6\frac{3}{4}$ and $q = 19\frac{1}{4}$, and substituting these values in (7) we find

$$z = -3\tfrac{1}{2}.$$

And because $k = z - \frac{1}{6}m$,

$$\therefore k = -3\tfrac{1}{4} - 2\tfrac{1}{2} = -6.$$

Substituting this value of k in (4) the equation becomes

$$x^2 - 6 = -\sqrt{3}\,.\,x + \sqrt{48}, \tag{8}$$

whence, transposing, completing the square and extracting the roots, we find

$$x = 2.8323801.$$

The roots of (3′) may be constructed by intersecting two plane curves, as follows:—

If we put $m' = \frac{1}{4}(m + 4)$, $n' = -\frac{1}{8}n$

and $r' = \frac{1}{8}\sqrt{4m^2 + n^2 + 32m + 16r + 64}$,

and construct a parabola whose parameter is 2; then, regarding the vertex of the parabola as the origin of co-ordinates, if we describe a circle, with radius r and co-ordinates of centre m', n', the perpendicular distances from the axis of the parabola to the intersections of the peripheries of the two curves will represent the roots of the equation.

Whatever values may be assigned to a, b, and c, as applied in this problem, the solution of (3′) will always give two imaginary roots, and two real, one positive and one negative. [*J. E. Hendricks.*]